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(NS5, D5, D3) bound state, OD3, OD5 limits and $SL(2, Z)$ duality

Indranil Mitra and Shibaji Roy

Theory Division, Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Calcutta-700 064, India

E-mails: indranil, roy@tnp.saha.ernet.in

ABSTRACT

We generalize the non-threshold bound state in type IIB supergravity of the form (NS5, D5, D3) constructed by the present authors (in hep-th/0011236) to non-zero asymptotic value of the axion (χ_0). We identify the decoupling limits corresponding to both the open D3-brane theory and open D5-brane theory for this supergravity solution as expected. However, we do not find any non-commutative Yang-Mills theory (NCYM) limit for this solution. We then study the $SL(2, Z)$ duality symmetry of type IIB theory for both OD3-limit and OD5-limit. We find that for OD3 theory, a generic $SL(2, Z)$ duality always gives another OD3-theory irrespective of the value of χ_0 being rational or not. This indicates that OD3-theory is self-dual. But, under a special set of $SL(2, Z)$ transformations OD3-theory goes over to a 5+1 dimensional NCYM theory, where these two theories are not necessarily related by strong-weak duality symmetry. On the other hand, for OD5-theory, a generic $SL(2, Z)$ duality gives another OD5-theory if χ_0 is irrational, but when χ_0 is rational it gives the little string theory limit indicating that OD5-theory is S-dual to the type IIB little string theory. However, under a special set of $SL(2, Z)$ transformations for which χ_0 is irrational OD5-theory reduces to 5+1 dimensional SYM theory.

1 Introduction

In a previous paper [1], we have constructed various non-threshold bound state solutions of both type IIB and type IIA supergravities of the type (NS5, Dp) (with $0 \leq p \leq 5$) and (NS5, D$(p+2)$, Dp) (with $0 \leq p \leq 3$) by applying a series of T- and S-dualities to the known (q, p) 5-brane solution of type IIB supergravity[1]. One of the motivations for constructing such solutions is to look at the world-volume theories of NS5-branes in the presence of various D-branes (or various RR electric gauge fields). In [3], it was argued that the world-volume theory of NS5-branes in the presence of a near critical RR $(p+1)$-form electric gauge field gives a non-gravitational and non-local theory called an open Dp-brane theory in a special low energy limit (decoupling limit) known as the ODp-limit. The (NS5, Dp) brane supergravity solution in this decoupling limit describes the supergravity dual of ODp-theories. These theories are analogous to world-volume theories of Dp-branes in the presence of near critical electric fields (NCOS theory) [4, 5] and world-volume theory of M5-brane in near critical electric 3-form gauge field (OM theory) [3, 6] and contains fluctuating light open Dp-branes in the world-volume of NS5-branes decoupled from gravity.

Since starting from these (NS5, Dp) bound state solutions it is possible to construct various other bound states containing NS5-branes and several different D-branes by applying a series of T- and S-dualities, it is natural to ask what kind of theories do they correspond to in the decoupling limit. Some of the cases have been studied in [7, 8]. In this paper we consider a specific case, namely, the (NS5, D5, D3) non-threshold bound state solution of type IIB supergravity. This is an $SL(2, Z)$ invariant bound state of type IIB theory and the supergravity solution for this state has been constructed in [1] for zero asymptotic value of the axion. We rewrite this solution for the non-zero asymptotic value of the axion (χ_0). This solution can also be regarded as NS5-branes in the presence of a 6-form and a 4-form RR electric gauge fields. There are m NS5-branes, n D5-branes and p D3-branes per $(2\pi)^2\alpha'$ of two codimensional area of NS5 (or D5)-branes in this bound state and preserves half of the space-time supersymmetries of string theory. We then identify both the open D3-brane limit and the open D5-brane limit for this supergravity solution. For the former, the 4-form approaches the critical value, whereas, for the latter the 6-form gauge field approaches the critical value. In this paper we only concentrate on the supergravity solution in the decoupling limit and also instead of writing the RR 6-form gauge field we write its Poincare dual. The existence of OD3- and OD5-limits for this bound state solution may not be surprising, however, to our surprise, we do not find any NCYM-limit in this $SL(2, Z)$ invariant solution. Since, (NS5, D3) state goes over to (D5, D3) state under S-duality, it was argued in [3], that the strong coupling limit of OD3-theory is the (5+1)-dimensional NCYM. So, it is surprising that no NCYM-limit exists for (NS5, D5, D3) state. However, it can be easily checked that when $m = 0$ (i.e. when NS5-branes are absent), the OD3-limit reduces exactly to the NCYM-limit [9].

[1]Some of these solutions are also considered in [2] from a different approach.

We then study the $SL(2,Z)$ transformation[2] on both the OD3-limit and the OD5-limit. We find that under a generic $SL(2,Z)$ transformation OD3-limit always gives another OD3-limit irrespective of whether χ_0 is rational or not. Since even for rational χ_0, we get another OD3-limit, we conclude that OD3-theory is self-dual. In other words, strongly coupled OD3-theory is related to the weakly coupled OD3-theory with different set of parameters related by S-duality. This is in accord with recent observation made in [14, 15], where it was emphasized that since 5+1 dimensional NCYM is non-renormalizable, so it can not be obtained from OD3-theory by S-duality. Thus OD3-theory must be self-dual and is the UV completion of the 5+1 dimensional NCYM. But, we find that under a special circumstance $SL(2,Z)$ duality on OD3-limit can give rise to NCYM-limit. If χ_0 is irrational, then these two theories are not related by strong-weak duality symmetry. For a particular value of rational χ_0, they are indeed related by strong-weak duality. Actually, what happens here is that under this special set of $SL(2,Z)$ transformations the transformed charge of NS5-brane vanishes. Therefore, the OD3-limit in the transformed solution reduces to NCYM-limit as we mentioned earlier. However, for OD5-limit, we find that when χ_0 is irrational a generic $SL(2,Z)$ transformation gives another OD5-limit with different parameters. But when χ_0 is rational OD5-limit gives us precisely the little string theory [16, 17, 18, 19, 20] limit. Thus we conclude that under the S-duality of type IIB theory OD5-theory goes over to little string theory. As in the case of OD3-limit here also we find that under a special set of $SL(2,Z)$ transformations for which χ_0 is irrational, the OD5-theory goes over to 5+1 dimensional SYM-theory [9].

This paper is organized as follows. In section 2, we give the (NS5, D5, D3) supergravity solution for non-zero asymptotic value of the axion. In section 3, we discuss the OD3- and OD5-limits for this solution. The $SL(2,Z)$ transformation is discussed in section 4. Finally, in secton 5, we present our conclusion.

2 (NS5, D5, D3) solution for non-zero χ_0

The non-threshold bound state solution of the type (NS5, D5, D3) of type IIB supergravity was constructed in [1] and has the form:

$$
\begin{aligned}
ds^2 &= H^{1/2}H''^{1/2}\left[H^{-1}\left(-dx_0^2+dx_1^2+\cdots+dx_3^2\right)+H'^{-1}\left(dx_4^2+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_b} &= g_s H'^{-1/2}H''\\
\chi &= \frac{\tan\psi}{g_s}(H''^{-1}-1)\\
B^{(b)} &= 2m\alpha'\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2+\tan\varphi\sin\psi H'^{-1}dx^4\wedge dx^5\\
A^{(2)} &= 2n\alpha'\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2-\frac{\cos\psi}{g_s}\tan\varphi H'^{-1}dx^4\wedge dx^5
\end{aligned}
$$

[2] $SL(2,Z)$ transformation on the various decoupling limits of (F, D1, D3) bound state has been studied in [10, 11, 12, 13].

$$
\begin{aligned}
A^{(4)} &= -p\alpha' H'^{-1} \sin^2\theta \cos\phi_1 dx^4 \wedge dx^5 \wedge d\theta \wedge d\phi_2 \\
&\qquad -\frac{\sin\varphi}{g_s} H^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3
\end{aligned} \tag{1}
$$

In the above $r = \sqrt{x_6^2 + x_7^2 + x_8^2 + x_9^2}$ and $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi_1^2 + \sin^2\theta \sin^2\phi_1 d\phi_2^2$ is the line element of the unit 3-sphere transverse to the 5-branes. $g_s = e^{\phi_{b0}}$ is the string coupling constant, $B^{(b)}$ and $A^{(2)}$ denote the NSNS and RR two-form potentials. χ is the RR scalar and $A^{(4)}$ is the RR 4-form gauge field whose field strength is self-dual. The harmonic functions H, H' and H'' are given as,

$$
\begin{aligned}
H &= 1 + \frac{Q_5}{r^2} \\
H' &= 1 + \frac{\cos^2\varphi Q_5}{r^2} \\
H'' &= 1 + \frac{\cos^2\varphi \cos^2\psi Q_5}{r^2}
\end{aligned} \tag{2}
$$

where the angles $\cos\varphi$, $\cos\psi$ and the charge Q_5 are defined as

$$
\begin{aligned}
\cos\varphi &= \frac{(m^2 + n^2 g_s^2)^{1/2}}{[m^2 + (p^2 + n^2) g_s^2]^{1/2}} \\
\cos\psi &= \frac{m}{(m^2 + n^2 g_s^2)^{1/2}} \\
Q_5 &= [m^2 + (p^2 + n^2) g_s^2]^{1/2} \alpha'
\end{aligned} \tag{3}
$$

Here m is the number of NS5-branes n is the number of D5-branes and p is the number of D3-branes per $(2\pi)^2\alpha'$ of two codimensional area of 5-branes.

Note here that since the harmonic functions in (2) approaches unity asymptotically, so the string metric in (1) becomes Minkowskian in this limit. Also, $e^{\phi_b} \to e^{\phi_{b0}}$ and $\chi \to 0$ asymptotically. In order to obtain this solution for non-zero asymptotic value of the axion (χ_0), we make an $SL(2,R)$ transformation by the matrix

$$
\Lambda = \begin{pmatrix} 1 & \chi_0 \\ 0 & 1 \end{pmatrix} \tag{4}
$$

The solution then takes the form:

$$
\begin{aligned}
ds^2 &= H^{1/2} H''^{1/2} \left[H^{-1} \left(-dx_0^2 + dx_1^2 + \cdots + dx_3^2 \right) + H'^{-1} \left(dx_4^2 + dx_5^2 \right) + dr^2 + r^2 d\Omega_3^2 \right] \\
e^{\phi_b} &= g_s H'^{-1/2} H'' \\
\chi &= \frac{\tan\psi}{g_s} (H''^{-1} - 1) + \chi_0 \\
B^{(b)} &= 2m\alpha' \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 + \tan\varphi \sin\psi H'^{-1} dx^4 \wedge dx^5 \\
A^{(2)} &= 2n\alpha' \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 - \left(\frac{\cos\psi}{g_s} + \chi_0 \sin\psi \right) \tan\varphi H'^{-1} dx^4 \wedge dx^5
\end{aligned} \tag{5}
$$

and $A^{(4)}$ retains its form as given in (1) since it is $SL(2,R)$ invariant. The harmonic functions retain their form as given in (2), but now the angles and the charge Q_5 are given as,

$$\begin{aligned}
\cos\varphi &= \frac{[m^2+(n+\chi_0 m)^2 g_s^2]^{1/2}}{[m^2+(p^2+(n+\chi_0 m)^2)g_s^2]^{1/2}} \\
\cos\psi &= \frac{m}{[m^2+(n+\chi_0 m)^2 g_s^2]^{1/2}} \\
Q_5 &= [m^2+(p^2+(n+\chi_0 m)^2)g_s^2]^{1/2}\alpha'
\end{aligned} \tag{6}$$

Note here that in terms of angles Q_5 in (6) can be written as $Q_5 = (m\alpha')/(\cos\varphi\cos\psi)$. Therefore, the harmonic functions in eq.(2) take the forms:

$$\begin{aligned}
H &= 1+\frac{m\alpha'}{\cos\varphi\cos\psi r^2} \\
H' &= 1+\frac{m\alpha'\cos\varphi}{\cos\psi r^2} \\
H'' &= 1+\frac{m\alpha'\cos\varphi\cos\psi}{r^2}
\end{aligned} \tag{7}$$

We will use these forms later.

From eq.(6) we deduce the following quantization conditions:

$$\begin{aligned}
\frac{n}{m} &= \frac{\tan\psi}{g_s} - \chi_0 \\
\frac{p}{m} &= \frac{\tan\varphi}{g_s\cos\psi}
\end{aligned} \tag{8}$$

Note here that under a general $SL(2,Z)$ transformation by the matrix

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{9}$$

where a, b, c, d are integers with $ad-bc=1$, Q_5, $\cos\varphi$, H and H' are invariant. But $\cos\psi$ and H'' change as

$$\begin{aligned}
\cos\hat{\psi} &= \frac{\left[\frac{c}{g_s}\tan\psi - (c\chi_0+d)\right]}{\left[(c\chi_0+d)^2+\frac{c^2}{g_s^2}\right]^{1/2}}\cos\psi \\
\hat{H}'' &= 1+\frac{\cos^2\varphi\cos^2\hat{\psi}Q_5}{r^2}
\end{aligned} \tag{10}$$

We will consider the various decoupling limits for this solution in the next section.

3 OD3 and OD5 limits

(a) OD3 limit: The open D3-brane theory appears as a decoupling limit on the world-volume of NS5-branes in the presence of a near critical RR 4-form electric gauge field and was obtained in [3]. The corresponding supergravity dual [1, 21] is given as the decoupling limit of (NS5, D3) bound state solution of type IIB theory. The OD3-limit is given as the following:

$$\cos\varphi = \epsilon \longrightarrow 0 \tag{11}$$

keeping the following quantities fixed,

$$\alpha'_{\text{eff}} = \frac{\alpha'}{\epsilon}, \qquad u = \frac{r}{\epsilon \alpha'_{\text{eff}}}, \qquad G^2_{o(3)} = g_s \tag{12}$$

Note here that if we set $\cos\psi = 1$ and $\chi_0 = 0$, then the solution given in (5) reduces to (NS5, D3) solution [1]. However, when D5-branes are also present OD3-limit is again the same as in (11) and (12), but in addition we set

$$\cos\psi = l \text{ (finite)} \qquad \text{and} \qquad \chi_0 \neq 0 \tag{13}$$

In the above ϵ is a dimensionless parameter, $(\alpha'_{\text{eff}})^{3/2}$ corresponds to the finite inverse tension of open D3-brane and $G^2_{o(3)}$ is the coupling constant. In this limit the harmonic functions in (7) reduce to

$$\begin{aligned} H &= \frac{1}{a^2\epsilon^2 u^2} \\ H' &= \frac{h'}{a^2 u^2} \\ H'' &= \frac{h''}{\tilde{a}^2 u^2} \end{aligned} \tag{14}$$

where $h' = 1 + a^2u^2$ and $h'' = 1 + \tilde{a}^2u^2$, with $a^2 = l\alpha'_{\text{eff}}/m$ and $\tilde{a}^2 = \alpha'_{\text{eff}}/(lm)$. So, the metric in (5) takes the following form,

$$ds^2 = \alpha' h''^{1/2}\left[-d\tilde{x}_0^2 + \sum_{i=1}^{3} d\tilde{x}_i^2 + h'^{-1}\sum_{j=4}^{5} d\tilde{x}_j^2 + \frac{m}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{15}$$

The finite coordinates in (15) are defined as

$$\begin{aligned} \tilde{x}_{0,1,2,3} &= \sqrt{\frac{l}{\alpha'_{\text{eff}}}}\, x_{0,1,2,3} \\ \tilde{x}_{4,5} &= \frac{\sqrt{l\alpha'_{\text{eff}}}}{\alpha'}\, x_{4,5} \end{aligned} \tag{16}$$

So, these are precisely the OD3-limit discussed in [3]. In this limit the dilaton and other gauge fields take the form:

$$
\begin{aligned}
e^{\phi_b} &= G^2_{o(3)} \frac{h''}{h'^{1/2}} \frac{l}{\tilde{a}u} \\
\chi &= -\frac{\sqrt{1-l^2}}{l} \frac{1}{G^2_{o(3)}} \frac{1}{h''} + \chi_0 \\
B^{(b)}_{\theta\phi_2} &= 2m\alpha' \sin^2\theta \cos\phi_1, \qquad B^{(b)}_{45} = \alpha' \frac{\sqrt{1-l^2}}{l} \frac{a^2u^2}{h'} \\
A^{(2)}_{\theta\phi_2} &= 2n\alpha' \sin^2\theta \cos\phi_1, \qquad A^{(2)}_{45} = -\alpha' \left(\frac{1}{G^2_{o(3)}} + \frac{\chi_0\sqrt{1-l^2}}{l} \right) \frac{a^2u^2}{h'} \\
A^{(4)}_{45\theta\phi_2} &= -\frac{\alpha'^3}{\alpha'_{\text{eff}}} p \frac{a^2u^2}{lh'} \sin^2\theta \cos\phi_1, \qquad A^{(4)}_{0123} = -\frac{\alpha'^2}{G^2_{o(3)}} \frac{a^2u^2}{l^2}
\end{aligned} \tag{17}
$$

Note that for $\frac{\tilde{a}u}{l} \ll 1$ (which implies that both $au \ll 1$ and $\tilde{a}u \ll 1$) i.e. in the IR region the supergravity description is valid (in this case the curvature $\alpha'\mathcal{R} \sim 1/m$ remains small for large enough m, the number of NS5-branes) if $\frac{\tilde{a}u}{l} \gg G^2_{o(3)}$. In this case $G^2_{o(3)} \ll 1$. However, this condition is not satisfied in the extreme IR region, where e^{ϕ_b} becomes large. In that case we have to go to the S-dual frame and we will describe this in the next section. In any case, in the IR region the OD3-theory flows to (5+1) dimensional SYM theory.

For $au \gg 1$, i.e. in the UV region, the string coupling $e^{\phi_b} = G^2_{o(3)} =$ fixed. So, when $G^2_{o(3)} \ll 1$, we have valid supergravity description and the metric in (15) reduces to that of ordinary D3-branes smeared in 4, 5 directions. However, for $G^2_{o(3)} \gg 1$, we have to go to the S-dual frame and will be discussed in the next section.

(b) OD5 limit: As in the previous case open D5-brane theory arises as a decoupling limit on NS5-branes when the RR 6-form electric gauge field approaches a critical value. The dual supergravity solution is obtained from (NS5, D5) bound state [1] in the decoupling limit. In the present case of (NS5, D5, D3) solution the OD5-limit is given in the following:

$$\cos\psi = \epsilon \to 0 \tag{18}$$

keeping the following quantities fixed,

$$\alpha'_{\text{eff}} = \frac{\alpha'}{\epsilon}, \qquad u = \frac{r}{\epsilon\alpha'_{\text{eff}}}, \qquad G^2_{o(5)} = \epsilon g_s \tag{19}$$

and also we have

$$\cos\varphi = \tilde{l} = \text{finite}, \qquad \text{and} \qquad \chi_0 \neq 0 \tag{20}$$

Under this decoupling limit the harmonic functions take the forms:

$$H = \frac{1}{a^2\epsilon^2u^2}$$

$$H' = \frac{1}{\tilde{a}^2\epsilon^2 u^2}$$
$$H'' = \frac{h''}{\tilde{a}^2 u^2} \tag{21}$$

where $h'' = 1 + \tilde{a}^2 u^2$, with $\tilde{a}^2 = \alpha'_{\text{eff}}/(m\tilde{l})$, $a^2 = \tilde{l}\alpha'_{\text{eff}}/m$.

The metric in (5) is now given by

$$ds^2 = \alpha' h''^{1/2}\left[-d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2 + \frac{m}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{22}$$

where the finite coordinates are defined as

$$\tilde{x}_{0,1,2,3} = \sqrt{\frac{\tilde{l}}{\alpha'_{\text{eff}}}}x_{0,1,2,3}, \qquad \tilde{x}_{4,5} = \frac{1}{\sqrt{\tilde{l}\alpha'_{\text{eff}}}}x_{4,5} \tag{23}$$

The dilaton, axion and other gauge fields in the decoupling limit are given as,

$$\begin{aligned}
e^{\phi_b} &= G^2_{o(5)}\frac{h''}{\tilde{a}u}\\
\chi &= -\frac{1}{G^2_{o(5)}}\frac{1}{h''} + \chi_0\\
B^{(b)}_{\theta\phi_2} &= 2m\alpha'\sin^2\theta\cos\phi_1, \qquad B^{(b)}_{45} = \frac{\alpha'^2}{m}\frac{\sqrt{1-\tilde{l}^2}}{\tilde{l}}u^2\\
A^{(2)}_{\theta\phi_2} &= 2n\alpha'\sin^2\theta\cos\phi_1, \qquad A^{(2)}_{45} = \frac{\alpha'^2}{m}\chi_0\frac{\sqrt{1-\tilde{l}^2}}{\tilde{l}}u^2\\
A^{(4)}_{45\theta\phi_2} &= -\frac{p}{m}\alpha'^3 u^2\sin^2\theta\cos\phi_1, \qquad A^{(4)}_{0123} = -\alpha'^3\frac{1}{m}\frac{\sqrt{1-\tilde{l}^2}}{\tilde{l}}u^2
\end{aligned} \tag{24}$$

This is precisely the OD5-limit discussed in [3]. However, instead of the RR 6-form electric gauge field, we have given here its Poincare dual. One can indeed check that the corresponding 6-form approaches the critical value given there in this limit.

It is clear from above that in the IR ($\tilde{a}u \ll 1$) the supergravity solution is valid if $\tilde{a}u \gg G^2_{o(5)}$. In that case, $G^2_{o(5)} \ll 1$. But in the extreme IR, this relation is not satisfied and we need to go to the S-dual frame. On the other hand, in the UV, the solution is valid if $\tilde{a}u \ll G^{-2}_{o(5)}$. This again is not satisfied in the extreme UV region and we need to go to the S-dual frame which we will discuss in the next section.

Thus we have obtained both the OD3-limit and OD5-limit for the (NS5, D5, D3) supergravity solution. This as we mentioned in the introduction is quite expected. However, contrary to our expectation, we do not find any NCYM limit for this solution. We just like to point out that the OD3-limit discussed in this section takes the form of NCYM limit when we set 'm' the number of NS5-branes exactly to zero. Since $m = 0$, $\chi_0 = 0$, implies

$\cos\psi = 0$, so the harmonic function $H'' = 1$. Also, note that since $m/\cos\psi = ng_s$, so, the other two harmonic functions in the limit (11) and (12) reduce to

$$
\begin{aligned}
H &= 1 + \frac{ng_s\alpha'}{\cos\varphi r^2} \to \frac{\tilde{b}^2}{a^2u^2\alpha'^2}\\
H' &= 1 + \frac{ng_s\alpha'\cos\varphi}{r^2} = \frac{h}{a^2u^2}
\end{aligned}
\tag{25}
$$

where we have defined $h = 1 + a^2u^2$ with $a^2 = \tilde{b}/(ng_s)$. Note that here we are interpreting $\alpha'_{\rm eff} = \tilde{b}$ as the non-commutativity parameter and the Yang-Mills coupling $g^2_{NCYM} = (2\pi)^3 g_s =$ fixed. With these forms of the harmonic functions the metric and the dilaton take precisely the same form as the dual of NCYM theory [8] :

$$
ds^2 = \alpha'\left[\frac{u}{R}\left(\left(-d\tilde{x}_0^2 + \sum_{i=1}^{3} d\tilde{x}_i^2\right) + h^{-1}\sum_{j=4}^{5} d\tilde{x}_j^2\right) + \frac{R}{u}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{26}
$$

where $R = \tilde{b}/a$ and the fixed coordinates are

$$
\tilde{x}_{0,1,2,3} = x_{0,1,2,3}; \qquad \tilde{x}_{4,5} = \frac{\tilde{b}}{\alpha'}x_{4,5} \tag{27}
$$

and

$$
e^{\phi_b} = g_s\left(\frac{u}{R}\right)\frac{\tilde{b}}{h^{1/2}} \tag{28}
$$

Since NCYM limit is nothing but OD3-limit of (NS5, D5, D3) solution with $m = 0$, so it is not clear whether there exists an NCYM limit for this solution (for $m \neq 0$) independent of OD3-limit.

4 $SL(2, Z)$ transformation

Type IIB string theory as well as its low energy limit, the corresponding supergravity theory, are well-known to possess an $SL(2, Z)$ duality symmetry. Since the (NS5, D5, D3) solution is $SL(2, Z)$ invariant, we would like to know what happens to the OD3-limit and OD5-limit under an $SL(2, Z)$ transformation. Under a general $SL(2, Z)$ transformation by the matrix given in (9) the various fields of type IIB supergravity transform as:

$$
\begin{aligned}
&g^E_{\mu\nu} \to g^E_{\mu\nu}, \qquad \lambda \to \frac{a\lambda + b}{c\lambda + d}, \qquad \begin{pmatrix} B^{(b)} \\ A^{(2)} \end{pmatrix} \to \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}\begin{pmatrix} B^{(b)} \\ A^{(2)} \end{pmatrix}\\
&A^{(4)} \to A^{(4)}
\end{aligned}
\tag{29}
$$

where $g^E_{\mu\nu}$ denotes the Einstein metric and $\lambda = \chi + ie^{-\phi_b}$. Under (29) the axion and the dilaton transform as

$$
\begin{aligned}
\hat{\chi} &= \frac{(a\chi + b)(c\chi + d) + ace^{-2\phi_b}}{|c\lambda + d|^2}\\
e^{\hat{\phi}_b} &= |c\lambda + d|^2 e^{\phi_b}
\end{aligned}
\tag{30}
$$

Since the Einstein metric $g^E_{\mu\nu} = e^{-\phi_b/2} g_{\mu\nu}$ (where $g_{\mu\nu}$ is the string metric) remains invariant under $SL(2,Z)$ transformation so, the string metric would transform as

$$d\hat{s}^2 = |c\lambda + d| ds^2 \tag{31}$$

If we now insist that the transformed metric should asymptotically be Minkowskian then the transformed metric would be given as

$$d\hat{s}^2 = \frac{|c\lambda + d|}{\left[(c\chi_0 + d)^2 + \frac{c^2}{g_s^2}\right]^{1/2}} ds^2 \tag{32}$$

We will show in the following how the metric and the dilaton would transform under the $SL(2,Z)$ for both the OD3-limit and OD5-limit. The transformation of the other gauge fields can be obtained in a straightforward manner.

(a) *$SL(2,Z)$ transformation and OD3-limit:* We would like to point out that the numerator appearing in the transformation of the angle $\cos\psi$ in (10) may vanish for a particular choice of a set of $SL(2,Z)$ transformations because of the quantization condition given in (8). More precisely, we note that since $\tan\psi/g_s - \chi_0 = n/m$ is a rational number, so when $c = m$ and $d = n$,

$$\frac{c}{g_s}\tan\psi - (c\chi_0 + d) = 0 \tag{33}$$

In that case, $\cos\hat{\psi} = 0$ and $\hat{H}'' = 1$. As it is clear from the discussion in section 3, the OD3-limit in this case would reduce to the NCYM-limit. So, in the following discussion we would consider the two cases ($c\tan\psi/g_s - (c\chi_0 + d) \neq 0$ and $= 0$) separately.

(i) $c\chi_0 + d \neq c\tan\psi/g_s$:

From the form of χ and e^{ϕ_b} of OD3-limit given in (17) we find that

$$|c\lambda + d| = \frac{\tilde{h}''^{1/2}}{h''^{1/2}} \tag{34}$$

where

$$\tilde{h}'' = \left(\frac{c}{G^2_{o(3)}}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)^2 + \tilde{a}^2 u^2 \left(\frac{c^2}{G^4_{o(3)}} + (c\chi_0 + d)^2\right) \tag{35}$$

We thus find from (32) and (15) that the transformed metric has the form

$$d\hat{s}^2 = \alpha' \frac{\tilde{h}''^{1/2}}{\left[(c\chi_0 + d)^2 + \frac{c^2}{g_s^2}\right]^{1/2}} \left[-d\tilde{x}_0^2 + \sum_{i=1}^{3} d\tilde{x}_i^2 + h'^{-1}\sum_{j=4}^{5} d\tilde{x}_j^2 + \frac{m}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{36}$$

By writing $\tilde{h}''$ as

$$\begin{aligned}
\tilde{h}'' &= \left(\frac{c}{G^2_{o(3)}}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)^2 (1 + \hat{a}^2 u^2) \\
&= \left(\frac{c}{G^2_{o(3)}}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)^2 \hat{h}''
\end{aligned} \tag{37}$$

where

$$\hat{a}^2 = \frac{\left[\frac{c^2}{G_{o(3)}^4} + (c\chi_0 + d)^2\right]}{\left(\frac{c}{G_{o(3)}^2}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)^2}\tilde{a}^2 \tag{38}$$

we can write the metric in (36) precisely in the same form as that of the OD3-limit in (15) i.e.

$$d\hat{s}^2 = \alpha' \hat{h}''^{1/2}\left[-d\hat{x}_0^2 + \sum_{i=1}^{3} d\hat{x}_i^2 + h'^{-1}\sum_{j=4}^{5} d\hat{x}_j^2 + \frac{\hat{m}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{39}$$

where

$$\hat{m} = \frac{\left(\frac{c}{G_{o(3)}^2}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)}{\left[(c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^4}\right]^{1/2}} m \tag{40}$$

and also we have rescaled the coordinates as

$$\hat{x}_{0,1,\ldots,5} = \frac{\left(\frac{c}{G_{o(3)}^2}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)^{1/2}}{\left[(c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^4}\right]^{1/4}}\tilde{x}_{0,1,\ldots,5} \tag{41}$$

Note that (38) gives the transformation of $\tilde{a}^2 = \alpha'_{\rm eff}/(ml)$ under $SL(2,Z)$, however, $a^2 = l\alpha'_{\rm eff}/m$ remains invariant.

The form of the dilaton can be obtained from (30) and (17) as,

$$e^{\hat{\phi}_b} = \hat{G}_{o(3)}^2 \frac{\hat{h}''}{h'^{1/2}}\frac{\hat{l}}{\hat{a}u} \tag{42}$$

Where the coupling constant of the $SL(2,Z)$ transformed OD3-theory is given by

$$\hat{G}_{o(3)}^2 = \frac{1}{G_{o(3)}^2}\left[(c\chi_0 + d)^2 G_{o(3)}^4 + c^2\right] \tag{43}$$

and

$$\hat{l} = \cos\hat{\psi} = \frac{\left(\frac{c}{G_{o(3)}^2}\frac{\sqrt{1-l^2}}{l} - (c\chi_0 + d)\right)}{\left[(c\chi_0 + d)^2 + \frac{c^2}{G_{o(3)}^4}\right]^{1/2}}\cos\psi \tag{44}$$

We note that the metric and the dilaton have the same forms as those of the OD3-limit obtained in eqs.(15) and (17) in the previous section. From (35), it is clear that no matter whether $c\chi_0 + d \neq 0$ (χ_0 is irrational) or $c\chi_0 + d = 0$ (χ_0 is rational) the forms of the metric and the dilaton in (36) and (42) are always the same as those of the original OD3-limit. The gauge fields of the $SL(2,Z)$ transformed OD3-theory can also be obtained from eqs.(29). We thus conclude that OD3-theory is self-dual. For rational χ_0, the metric and the dilaton are given by the same expressions as in (39) and (42) with a simpler form of $\tilde{h}''$, $\hat{a}^2$, $\hat{m}$, $\hat{x}_{0,1,\ldots,5}$ and $\hat{G}^2_{o(3)}$. They are given in the following

$$
\begin{aligned}
\tilde{h}'' &= \frac{c^2}{G^4_{o(3)}} \frac{1-l^2}{l^2}(1+\hat{a}^2u^2) \\
\hat{a}^2 &= \frac{l^2}{1-l^2}\tilde{a}^2, \qquad \hat{m} = \frac{\sqrt{1-l^2}}{l}m \\
\hat{x}_{0,1,\ldots,5} &= \frac{(1-l^2)^{1/4}}{\sqrt{l}}\tilde{x}_{0,1,\ldots,5}, \qquad \hat{G}^2_{o(3)} = \frac{c^2}{G^2_{o(3)}}
\end{aligned}
\tag{45}
$$

These are the various variables and parameters for the supergravity dual of S-dual OD3-theory. We note that since $a^2 = l\alpha'_{\text{eff}}/m$ remains invariant and l, m transform exactly in the same way (see (40) and (44)), so, α'_{eff} is invariant.

(ii) $c\chi_0 + d = c\tan\psi/g_s$:

From (35) we notice that when the above condition is satisfied we get

$$
\tilde{h}'' = \frac{\tilde{a}^2u^2}{l^2}\frac{c^2}{G^4_{o(3)}}
\tag{46}
$$

Then the metric in (31) has exactly the same form as in (26). Also, from (30), the dilaton is given by,

$$
e^{\hat{\phi}_b} = \hat{g}_s\left(\frac{u}{R}\right)\frac{\tilde{b}}{h'^{1/2}}
\tag{47}
$$

where $\hat{g}_s = m^2/(l^2g_s)$ and $h' = 1 + a^2u^2$, with $a^2 = \alpha'_{\text{eff}}/(ng_s) = \tilde{b}/(ng_s)$, $R = \tilde{b}/a$. This is precisely the NCYM limit, we discussed in section 3, with $\tilde{b}$ as the non-commutativity parameter and $g^2_{NCYM} = (2\pi)^3\hat{g}_s$, the Yang-Mills coupling. Thus we find NCYM-limit from a special set of $SL(2,Z)$ transformations of OD3-limit. Note here that since $c\chi_0 + d \neq 0$ in general so, OD3-theory and the NCYM-theory are not necessarily related by strong-weak duality symmetry as it is clear from $g_s\hat{g}_s = m^2/l^2 > 1$. On the other hand if χ_0 is chosen such that $c\chi_0 + d = 0$ (this can only be true if $\chi_0 = -n/m$), then we have $\sin\psi = 0$ i.e. there are no D5-branes in our original solution. In this case OD3-theory will be related to NCYM-theory by strong-weak duality symmetry as discussed in [3].

(b) $SL(2,Z)$ transformation and OD5-limit: As in the previous subsection we would first consider $c\tan\psi/g_s - (c\chi_0 + d) \neq 0$ and then consider the case when it is zero.

(i) $c\chi_0 + d \neq c\tan\psi/g_s$:

Proceeding exactly in the same way as in subsection (a) we first find the form of $|c\lambda + d|$ from the expressions of χ and e^{ϕ_b} in (24) as,

$$
|c\lambda + d| = \frac{\tilde{h}''^{1/2}}{h''^{1/2}}
\tag{48}
$$

where

$$\tilde{h}'' = \left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]^2 + \tilde{a}^2 u^2 (c\chi_0 + d)^2 \tag{49}$$

Note that since the form of $\tilde{h}''$ changes completely for irrational χ_0 and rational χ_0, we study these two cases separately.

Irrational χ_0

When $c\chi_0 + d$ is not equal to zero, we find from (32) and (22) that the transformed metric takes the form:

$$d\hat{s}^2 = \alpha' \frac{\tilde{h}''^{1/2}}{(c\chi_0 + d)} \left[-d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2 + \frac{m}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{50}$$

Redefining

$$\begin{aligned} \tilde{h}'' &= \left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]^2 (1 + \hat{a}^2 u^2) \\ &= \left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]^2 \hat{h}'' \end{aligned} \tag{51}$$

where $\hat{h}'' = 1 + \hat{a}^2 u^2$ and

$$\hat{a}^2 = \frac{(c\chi_0 + d)^2}{\left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]^2} \tilde{a}^2 \tag{52}$$

we can rewrite the metric in (50) exactly in the same form as that of OD5-limit, namely,

$$d\hat{s}^2 = \alpha' \hat{h}''^{1/2} \left[-d\hat{x}_0^2 + \sum_{i=1}^{5} d\hat{x}_i^2 + \frac{\hat{m}}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \tag{53}$$

where

$$\hat{m} = \frac{\left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]}{(c\chi_0 + d)} m \tag{54}$$

we have also redefined the coordinates as,

$$\hat{x}_{0,1,\ldots,5} = \frac{\left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right]^{1/2}}{(c\chi_0 + d)^{1/2}} \tilde{x}_{0,1,\ldots,5} \tag{55}$$

The transformed dilaton can be obtained from (30) and (24) as,

$$e^{\hat{\phi}_b} = \hat{G}^2_{o(5)} \frac{\hat{h}''}{\hat{a}u} \tag{56}$$

where

$$\hat{G}^2_{o(5)} = (c\chi_0 + d)\left[\frac{c}{G^2_{o(5)}} - (c\chi_0 + d)\right] G^2_{o(5)} \tag{57}$$

Here we note that since $\cos\varphi = \tilde{l} = a/\tilde{a} =$ invariant under $SL(2,Z)$, so both $\tilde{a}$ and a must transform in the same way. The transformation of $\tilde{a}$ is given in (52). However, $\cos\psi = \epsilon = m/(\sqrt{m^2 + (n+\chi_0 m)^2 g_s^2})$ is not $SL(2,Z)$ invariant, but it must transform as m i.e.

$$\cos\hat{\psi} = \hat{\epsilon} = \frac{\left[\frac{c}{G_{o(5)}^2} - (c\chi_0 + d)\right]}{(c\chi_0 + d)} \cos\psi \tag{58}$$

Thus we observe from (53) and (56) that the transformed metric and the dilaton have exactly the same form as those of OD5-limit obtained in (22) and (24). Also since $\tilde{a}^2 = \alpha'_{\rm eff}/(m\tilde{l})$, so the effective tension of the $SL(2,Z)$ transformed OD5-theory would be given as,

$$\hat{\alpha}'_{\rm eff} = \frac{(c\chi_0 + d)}{\left[\frac{c}{G_{o(5)}^2} - (c\chi_0 + d)\right]} \alpha'_{\rm eff} \tag{59}$$

The forms of the other gauge fields of the $SL(2,Z)$ dual OD5-theory can be obtained from (29).

<u>*Rational* χ_0</u>

Now if χ_0 is rational i.e. $c\chi_0 + d = 0$, then from (48) we have

$$|c\lambda + d| = \frac{\tilde{h}''^{1/2}}{h''^{1/2}} \tag{60}$$

where $\tilde{h}'' = c^2/G_{o(5)}^4$. So, the metric and the dilaton would now take the forms,

$$\begin{aligned} d\hat{s}^2 &= \left[-d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2 + \alpha'_{\rm eff}\frac{m}{u^2}\left(du^2 + u^2 d\omega_3^2\right)\right] \\ e^{\hat{\phi}_b} &= \frac{c^2}{G_{o(5)}^2}\frac{1}{\tilde{a}u} \end{aligned} \tag{61}$$

Here $G_{o(5)}^2$ is just a finite quantity which can be absorbed in $\tilde{a}$. We have also redefined the coordinates $\tilde{x}_{0,1,\ldots,5}$ by $\sqrt{\alpha'_{\rm eff}}\tilde{x}_{0,1,\ldots,5}$ and $\alpha'_{\rm eff} = \alpha'/\epsilon =$ finite. Note also that although $e^{\hat{\phi}_b}$ is finite, the coupling constant for the dual theory $\hat{G}_{o(5)}^2 = \hat{g}_s = (c^2/G_{o(5)}^2)\epsilon \to 0$. Eq.(61) represents precisely the supergravity dual of little string theory [9, 20]. We thus conclude that the S-dual of OD5-theory is the little string theory.

<u>(ii) $c\chi_0 + d = c\tan\psi/g_s$:</u>

From (49), we find that $\tilde{h}''$ in this case takes the form

$$\tilde{h}'' = \frac{c^2}{G_{o(4)}^4}\tilde{a}^2 u^2 \tag{62}$$

The metric in (32) and the dilaton (30) can then be calculated to give,

$$\begin{aligned} ds^2 &= \alpha'\left[\frac{u}{R}\left(-d\tilde{x}_0^2 + \sum_{i=1}^{5} d\tilde{x}_i^2\right) + \frac{R}{u}\left(du^2 + u^2 d\Omega_3^2\right)\right] \\ e^{\hat{\phi}_b} &= \frac{g_{YM}^2}{(2\pi)^3}\frac{u}{R} \end{aligned} \tag{63}$$

where we have defined $R = (2\pi)^3/(g_{YM}\sqrt{\hat{n}})$, $\hat{n}$ being the $SL(2,Z)$ transformed charge of D5-branes and the Yang-Mills coupling $g_{YM}^2 = (2\pi)^3 \hat{g}_s \alpha'/\tilde{l}$, with $\hat{g}_s = g_s c^2/G^4_{o(5)}$. We have also redefined the coordinates as, $\tilde{x}_{0,1,2,3} = x_{0,1,2,3}$ and $\tilde{x}_{4,5} = x_{4,5}/\tilde{l}$. This is precisely the supergravity dual of 5+1 dimensional SYM-theory [9]. So, we obtain 5+1 dimensional SYM-theory as a special set of $SL(2,Z)$ transformations of OD5-theory. Note here that in this case $c\chi_0 + d$ can not be equal to zero i.e. χ_0 remains irrational. We would also like to point out that in obtaining (63), we have expressed $\alpha'_{\rm eff} m$ in terms of the $SL(2,Z)$ transformed charge $\hat{n}$ of D5-branes. In fact, since $\alpha'_{\rm eff} m$ is $SL(2,Z)$ invariant, it can be expressed as $\hat{n}\hat{g}_s\alpha'$.

5 Conclusion

To summarize, we have studied in this paper the various decoupling limits of an $SL(2,Z)$ invariant bound state of the type (NS5, D5, D3) in type IIB supergravity. This solution can also be regarded as NS5-branes in the presence of both a 4-form and a 6-form RR electric gauge fields. In particular, we have identified an OD3-limit and an OD5-limit for this solution. In these decoupling limits (NS5, D5, D3) solution represent the supergravity dual of OD3-theory and OD5-theory respectively. We have mentioned that when NS5-branes are absent, the OD3-limit reduces to an NCYM limit. But we do not find an independent NCYM limit in the presence of NS5-branes. We then studied the $SL(2,Z)$ transformation of both OD3-limit and OD5-limit. The generic $SL(2,Z)$ transformation of OD3-limit always gives another OD3-limit with different set of parameters irrespective of whether the asymptotic value of axion is irrational or not. When the asymptotic value of the axion is rational the two OD3-theories are related to each other by strong-weak duality symmetry. We thus conclude that OD3-theory is self-dual. However, under a special set of $SL(2,Z)$ transformations we find that the OD3-limit reduces to NCYM-limit. But for these set of $SL(2,Z)$ transformations the transformed charge of NS5-brane vanishes. Thus it is not surprising that we get an NCYM-limit for these transformations as we have already mentioned in section 3. In this case when χ_0 is irrational, the OD3-theory and NCYM-theory are not related by strong-weak duality symmetry, however for a particular rational value of χ_0, they are indeed related by strong-weak duality symmetry and this case has already been studied in [3]. On the other hand, for OD5-limit, we find that when the asymptotic value of axion is irrational a generic $SL(2,Z)$ transformation gives another OD5-limit with different set of parameters characterizing the $SL(2,Z)$ transformed OD5-theory. But when χ_0 is rational OD5-limit reduces to little string theory limit. So, we conclude that OD5-theory and little string theory are related to each other by type IIB S-duality symmetry. But, as in OD3 case we find that under a special set of $SL(2,Z)$ transformations OD5-theory reduces to 5+1 dimensional ordinary SYM theory and χ_0 in this case remains irrational.

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